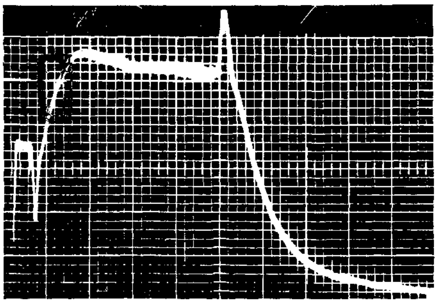

Fig 2 Temperature vs time profile of metal-oxidant mix at 250-psig environmental pressure

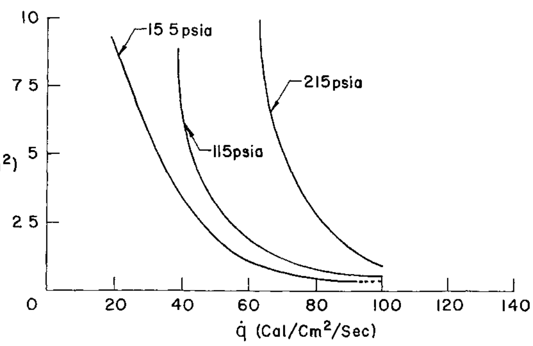

Fig 3 Approximate relationships of threshold ignition energy to flux at three pressure levels for a metal-oxidant mix at a consolidation pressure of 46,000 psi

"up-down" method[2] of statistical analysis can be employed to yield the q value estimate of the mean Since the shuttering mechanism delivers the energy in essentially a square wave form, the following relationship applies:

$$q = \dot{q}t \tag{1}$$

where $\dot{q}$ is the flux or rate at which the energy is delivered and t is the time The $\dot{q}$ range can be varied from about 15 to 125 cal/cm^2/sec, and the shuttering arrangement permits continuously variable, predetermined time increments between 5 and 2000 msec Calibration is obtained by means of a copper-constantan calorimeter

Samples ignited during a given analytical run are recorded by the radiation transducer previously described The ignition point with respect to time is clearly visible on the oscilloscope traces When the burning wave front reaches the end of the sample, the zirconium bead mix is ignited, thus producing an intense signature (see Fig 2) The burning time can then be measured, and since the sample length is known, the burning rate is easily determined Burning rate exponents obtained in this manner indicate close agreement with those obtained by the Crawford strand-burner method [3]

Since the $\dot{q}$ can be varied by the introduction of steel wire mesh screens near the shuttering mechanism, the manner in which the q value of a particular material varies with $\dot{q}$ can be easily ascertained [4] Figure 3 shows the q vs $\dot{q}$ relationship of a metal-oxidant mix at three environmental pressure levels

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Phase Shift in a Massless Vibrating Beam

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THIS note is a simple demonstration of a phase shift in a massless vibrating beam The mathematical model shown in Fig 1 consists of a massless cantilever beam with a concentrated mass at its free end being driven by a forcing function $F_0e^{i\omega t}$

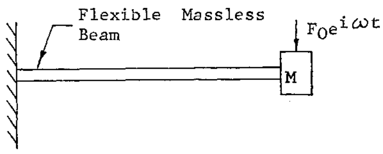

Fig 1 Mathematical model for the study of the phase shift

A free-body diagram in which the beam is represented as a series of elastic and viscous elements is shown in Fig 2 In general, the equation of motion for any section is

$$M\ddot{x}_n = -k_j(x_j - x_{j-1}) - c_j(\dot{x}_j - \dot{x}_{j-1}) + F_0e^{i\omega t} \tag{1}$$

where j represents the section being considered Consider the steady-state solution to Eq (1):

$$x_j = A_je^{i\omega t} \tag{2}$$

where the A_j are complex Values for A_j may be obtained

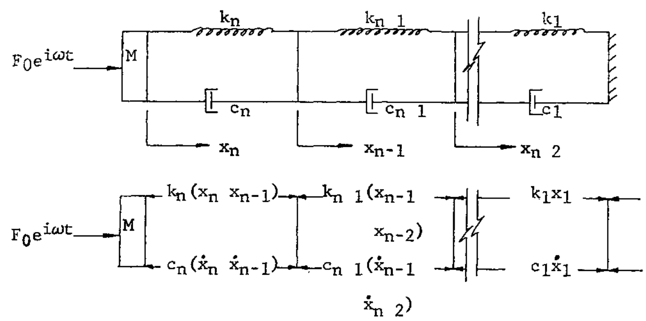

Fig 2 Free-body diagram of the mathematical model

by placing Eqs (2) into Eqs (1) Letting $A_j = r_je^{i\alpha_j}$, a ratio of successive amplitudes is found to be

$$\frac{A_j}{A_{j-1}} = \frac{r_j}{r_{j-1}}\, e^{i(\alpha_j - \alpha_{j-1})} \tag{3}$$

Using Eqs (1–3), the phase shift at any position along the beam may be formulated Hence, an arbitrary selection of $n = 3$ and $j = 2$ yields,

$$\tan(\alpha_2 - \alpha_1) = \frac{(c_1k_2 - c_2k_1)\omega}{k_2(k_1 + k_2) + \omega^2c_2(c_2 + c_1)} \tag{4}$$

For $c_1 = c_2$ and $k_1 = k_2$, Eq (4) reduces to $\tan(\alpha_2 - \alpha_1) = 0$, which implies that no phase difference exists between the different massless sections Physically, this represents a uniform beam where the damping is not a function of displace-

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ment In the case of a uniform beam where damping is a function of displacement, Eq (4) becomes

$$\tan(\alpha_2 - \alpha_1) = \frac{\omega k(c_1 - c_2)}{2k^2 + \omega^2 c_2(c_1 + c_2)} \qquad (5)$$

Hence, a massless beam may have a phase shift

Effect of Aerodynamic Damping on Flutter of Thin Panels

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THIS note discusses the role of aerodynamic damping in determining the flutter boundary for thin panels, primarily within the framework of piston theory aeiodynamics

In the early work on the flutter of unstressed, flat, rectangular panels, it was shown that the neglect of aerodynamic damping was a very good approximation for panels of moderate to large aspect ratios (see, e g , Refs 1 and 7) Thus, the flutter boundary could be described in terms of the now well-known parameter,

$$\lambda = 2qL^3/(M^2 - 1)^{1/2}D$$

The omission of aerodynamic damping was equivalent to setting the mass ratio

$$\mu = \rho L/\rho m h$$

equal to zero

Subsequent to this work, the approximation $\mu = 0$ has been used in determining the stability boundaries for numerous configurations Here we would like to note that there are situations where this approximation may lead to inaccurate results This is usually associated with flutter involving modes of nearly identical frequency but weak aerodynamic coupling

1 Cylindrical Shell

As discussed in Ref 2, for the cylindrical shell there are two types of flutter modes of interest One of these is a coupling of the first two axial modes at a large circumferential mode number For this type of flutter mode the approximation is still a good one (under the assumption of piston theory aerodynamics) The other flutter mode of interest is an axisymmetric coupling of two higher axial modes For this type of flutter mode aerodynamic damping has a very pronounced effect on the flutter boundary In addition, flutter involving other modes of nearly identical frequency, but weakly coupled aerodynamically, can be shown to be unimportant due to aerodynamic damping

2 Rectangulai Panels under Shearing Loads

Recently, Eisley and Luessen[3] considered the problem of a rectangular, flat panel under shearing loads These shearing loads lead to the merging or crossing of the natural frequencies associated with an otherwise unstressed panel, and flutter boundaries would be obtained with sharp minimums in the flutter dynamic pressure for certain values of the shearing loads for $\mu = 0$ The inclusion of aerodynamic damping removes these unrealistic "dips"

3 Rectangular Panel Aligned at an Arbitrary Angle to the Flow

As a final example, the case of a flat, rectangular panel aligned at an arbitrary angle to the airflow is considered This problem has been studied previously in Refs 3–5 Recent calculations by the present authors *including* aerodynamic damping have led to the results shown in Fig 1 This figure gives the flutter parameter λ as a function of flow angle φ for two values of μ/M; $\mu/M = 0\,1$ was chosen as being representative of practical values of the mass ratio As may be seen, the addition of aerodynamic damping changes the character of the curve markedly In particular, the rapid vaiiation of λ in the neighborhood of $\phi = 0$ is no longer present for $\mu/M = 0\,1$

Fortunately, the inclusion of the aerodynamic damping term in the piston theory (or quasi-steady theory) is relatively simple for panels of uniform mass distribution For these panels a nondimensional generalized eigenvalue may be identified as

$$\theta = K^2 - i[\lambda\mu/M]^{1/2}K \qquad (1)$$

where

$$K^2 = \rho m h L^4\omega^2/D$$

is a nondimensional, complex panel frequency Equating real and imaginary parts of Eq 1 and specifying the flutter condition, $K_{\text{im}} = 0$, gives

$$K_R = \pm(\theta_R)^{1/2} \qquad (2)$$

$$\mu/M = (1/\lambda)\,(\theta_I^2/\theta_R) \qquad (3)$$

Strictly speaking, for the quasi-steady theory, μ/M should be replaced by

$$[\mu/(M^2 - 1)^{1/2}]\,[(M^2 - 2)/(M^2 - 1)]^2$$

It is to be noted that θ is only a function of λ (for a given panel geometry) Thus the mathematical problem is reduced to finding $\theta = \theta(\lambda)$ After θ has been deteimined, Eqs (2) and (3) may be used to determine flutter frequency and mass ratio, respectively This is essentially the stability parabola concept of Movchan[6] and Houbolt[7] Viscous structural damping also may be included by an analogous procedure

Finally, it should be mentioned that there are other aerodynamic damping effects which piston theory is incapable of predicting Foremost among these is the possibility of single degiee of freedom flutter when the aerodynamic damping becomes negative at low supersonic Mach numbers For this type of instability, λ is no longer an appropriate parameter

Lock and Fung[8] have investigated this phenomenon experimentally and theoretically for a nominal two-dimensional panel Moie recent work on the subject has shown that it exists over a limited range in Mach number which decreases

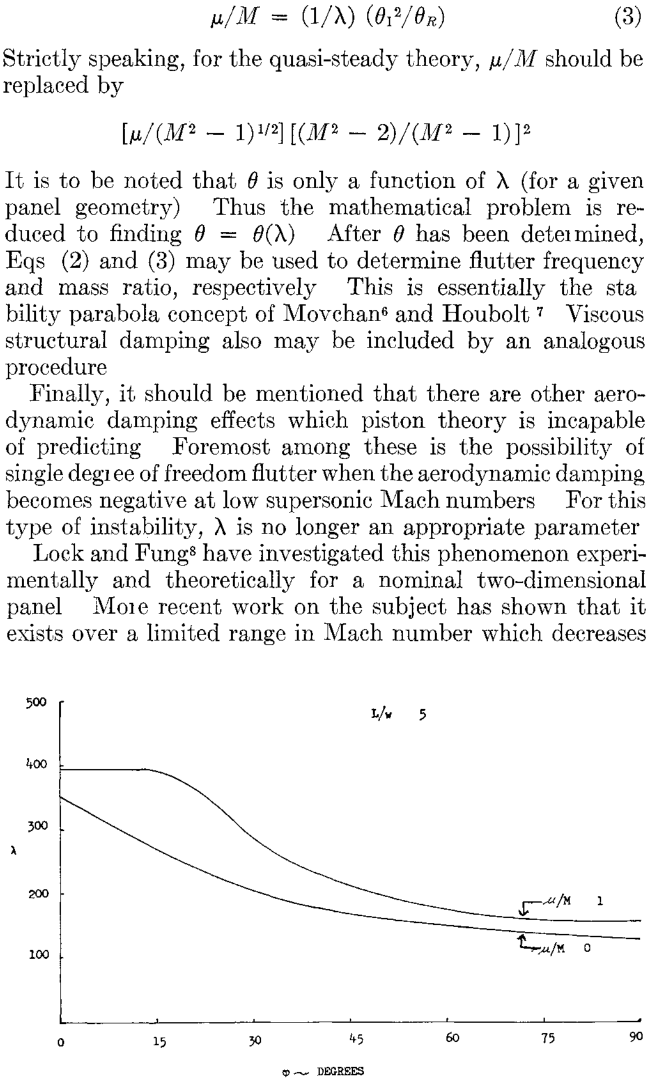

Fig 1 λ vs φ

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